The effect of velocity structure errors on double-difference earthquake location

A. Michelini[1]
Istituto Nazionale di Geofisica e Vulcanologia (INGV), Roma, Italy

A. Lomax
Anthony Lomax Scientific Software, Mouans-Sartoux, France

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[1] We show that relative earthquake location using double-difference methods requires an accurate knowledge of the velocity structure throughout the study region to prevent artifacts in the relative position of hypocenters. The velocity structure determines the ray paths between hypocenters and receivers. These ray paths, and the corresponding ray take-off angles at the hypocenters, determine the partial derivatives of travel time with respect to the hypocentral coordinates which form the inversion kernel that maps double-differences into hypocentral perturbations. Thus the large-scale velocity structure enters into the core of the double-difference technique. By employing a 1D layered model with sharp interfaces to perform double-difference inversion of synthetic data generated using a simple, 1D gradient model; we show that inappropriate choice of the velocity model, combined with unbalanced source-receiver distributions, can lead to significant distortion and bias in the relative hypocenter positions of closely spaced events. *INDEX TERMS:* 7215 Seismology: Earthquake parameters; 7230 Seismology: Seismicity and seismotectonics; 7260 Seismology: Theory and modeling; 8180 Tectonophysics: Tomography. **Citation:** Michelini, A., and A. Lomax (2004), The effect of velocity structure errors on double-difference earthquake location, *Geophys. Res. Lett.*, *31*, L09602, doi:10.1029/2004GL019682.

1. Introduction

[2] Accurate earthquake locations are of primary importance when studying the seismicity of a given area-they allow important inferences on the ongoing seismo-tectonics.

[3] In a recent paper, *Waldhauser and Ellsworth* [2000] (WE2000 hereafter) present an approach for determining high-precision, relative earthquake locations based on earlier work by *Got et al.* [1994]. More recently, *Zhang and Thurber* [2003] have performed 3-D earthquake tomography with both arrival time and double differences data using the algorithm of WE2000. *Wolfe* [2002] shows that the methods of *Jordan and Sverdrup* [1981], *Got et al.* [1994] and WE2000 are related, and compares their mathematical foundations and performance. Many studies using these location methods find dramatic features such as “streaks” in the hypocenter distributions [e.g., *Rubin et al.*, 1999; *Waldhauser et al.*, 1999; *Rubin*, 2002; *Schaff et al.*, 2002; *Waldhauser and Ellsworth*, 2002; *Chiaraluce et al.*, 2003]. Such results are of primary geophysical and geological importance and so their robustness must be firmly established. Here we show that use of an incorrect velocity model in relative location can lead to a distortion in the relative position of hypocenters that may, in some cases, result in artifacts such as apparent lineations.

[4] The relative location method of WE2000 uses an iterative linearized inversion of differences between same-phase travel-time residual for multiple events and is referred to as the *double-difference* method (DD). The core of any linear inversion is comprised of the mapping matrix, or kernel, of the inversion. In the DD case, the mapping matrix is composed of partial derivatives of travel times with respect to the hypocenter coordinates. In practice, these derivatives are directly related to the take-off angles, at the hypocenters, of the rays connecting the hypocenters and the receivers. Thus the inverse mapping of the DD data (double-differences of the residual travel times) into perturbations in relative event hypocenter locations and origin times is controlled by the ray take-off directions at the hypocenters; if these ray directions are significantly different from the true ray directions, then we may hypothesize that the information in the residual travel-time double-differences will be mapped incorrectly into hypocenter perturbations, and bias and error will be introduced.

[5] In WE2000, the DD inversion is performed using a 1D velocity model with discontinuities in velocity at layer interfaces. For a hypocenter above an interface, the rays leaving the source are up-going for nearby stations, but discontinuously change to steeply down-going for more distant stations, and there are no horizontal or moderately down-going rays (Figure 1b). A realistic crustal structure for a complex region such as Central California is unlikely to have extensive horizontal interfaces - the structure would be better approximated by a smooth model, with or without 3D, lateral variations. Then, depending on the velocity gradient, the rays leaving the source are up-going for nearby stations, and smoothly rotate to sub-horizontal to slightly down-going for more distant stations; the rays jump to steeply down-going only for stations beyond the Moho crossover distance (Figure 1a).

[6] Neither WE2000, *Zhang and Thurber* [2003], nor *Wolfe* [2002] examines or addresses explicitly error and bias in the ray directions and thus error and bias in the partial

[1]Previously at Istituto Nazionale di Oceanografia e di Geofisica Sperimentale-OGS, Trieste, Italy.

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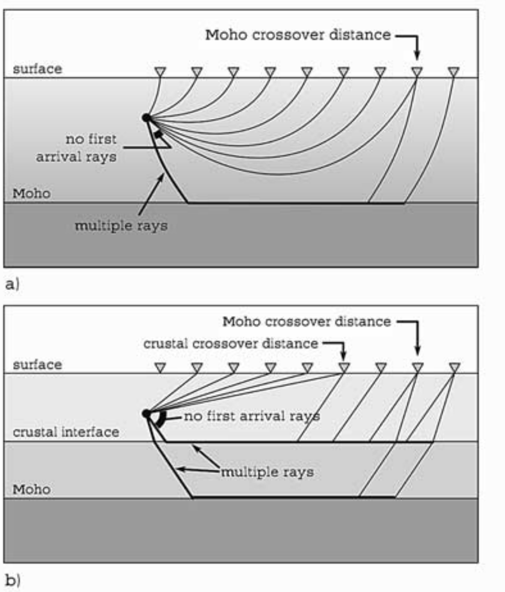

Figure 1. Cartoon showing the take-off angles for rays leaving the same hypocenter with two different velocity structures. a) The take-off angles for a 1D-gradient model with velocity increasing with depth. b) The same rays for a 1D-layered model with constant velocity layers.

derivatives that are the core of the inversion, though it is claimed that the DD technique minimizes errors due to large-scale un-modeled velocity structure.

[7] In this work, we test the hypothesis that velocity model errors can lead to significant bias and error in relative locations of earthquakes belonging to the same cluster (i.e., closely spaced events). We generate a synthetic data set of travel times using a 1-D, gradient velocity model for a realistic source receiver-geometry (Figure 2), and invert this data with the DD technique using an incorrect layered model and an incorrect smooth model (Figure 2). We compare and contrast the resulting relative locations and explain the differences through examination of the ray-take-off angles and other indicators.

2. Method

[8] The DD algorithm [*Waldhauser and Ellsworth*, 2000] is based on the differential travel time of either P or S times between two events. For two events *i* and *j*, and for the same station, *k*, we can write

$$dr_k^{ij} = r_k^i - r_k^j = \left(t^{obs} - t^{cal}\right)_k^i - \left(t^{obs} - t^{cal}\right)_k^j. \quad (1)$$

[9] Each travel-time residual *r* can be linearly related to the hypocentral perturbations Δx^i, so that

$$r_k^i = \frac{\partial t_k^i}{\partial \mathbf{x}} \Delta \mathbf{x}^i, \quad (2)$$

and it follows that

$$dr_k^{ij} = \frac{\partial t_k^i}{\partial \mathbf{x}} \Delta \mathbf{x}^i - \frac{\partial t_k^j}{\partial \mathbf{x}} \Delta \mathbf{x}^j. \quad (3)$$

[10] Equation (3) links the difference between residual times to the sought perturbations of the hypocentral parameters. Note that the DD minimization of dr_k^{ij} attempts to equalize, but not necessarily reduce, the residual times r_k^i, r_k^j, etc., at each station for closely located earthquakes. This is the fundamental difference that distinguishes DD event relocations from standard, single event location.

[11] Computationally, Equation (3) is set up in matrix form

$$\mathbf{dr} = \mathbf{Adx}, \quad (4)$$

and it is necessary to invert matrix **A** to estimate the adjustments to apply to the earthquake foci.

[12] To stabilize the inversion, we have added the possibility of regularizing the system of equations by constraining the solution to one or more reference events. This regularization scheme insures that a well-located reference event does not move from its initial position, whereas the other foci are left to adjust with respect to one another, while maintaining the reference event to control the absolute spatial location (i.e., it mimics the master event technique within the DD formalism).

[13] To verify the robustness of the results, the DD inversion has been performed both with the DD algorithm made public by *Waldhauser* [2001] and with codes developed independently by the authors. We have found that the results obtained were always consistent.

3. Test With Synthetic Data

[14] Our test involves a circular cluster of events lying on a N-S trending vertical plane. The diameter of the cluster is 1.0 km and consists of 21 foci (20 on the circle and one at

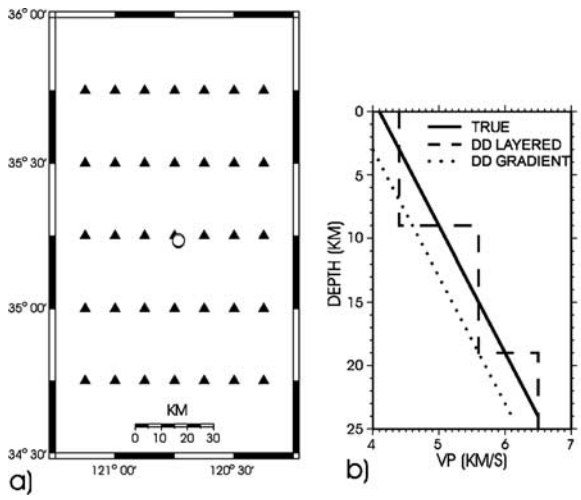

Figure 2. a) Source-receiver geometry used to generate the synthetic data set adopted in the tests. The solid triangles indicate the station used; the open circle shows the position of the circular cluster of events (1 km diameter) aligned on a vertical plane along the N-S direction. b) Velocity models used in our test: true velocity model used to generate the synthetic data (solid line); DD inversion models: layered (dashed line), and shifted linear gradient model (dotted line).

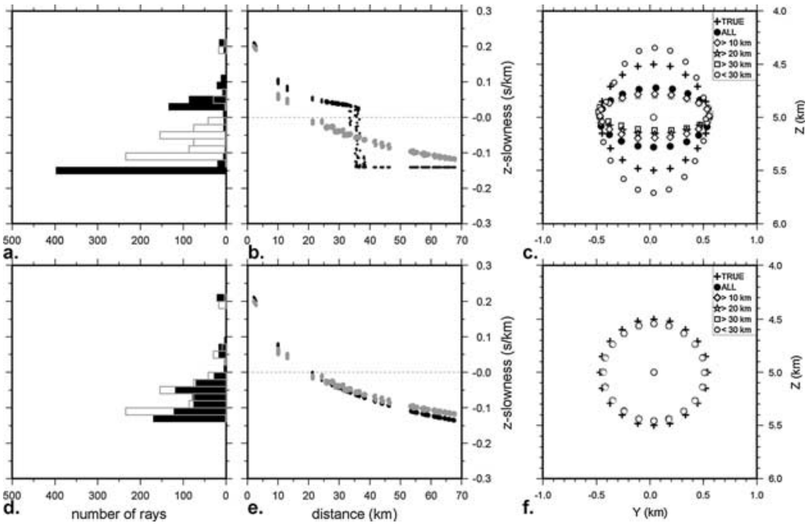

Figure 3. Results obtained with layered (top panels) and shifted linear gradient (bottom panels) velocity models. a) and d) Histograms showing the distribution of the true (light gray histogram) and adopted (black filled histogram) vertical slowness values entering into the mapping matrix of equation (3). b) and e) Distance versus vertical slowness plot that shows how the take off angle varies with receiver distance, light gray solid circles refer to the true model and black solid circles to those used in the double-difference inversion. Results obtained after application of DD to the layered c) and to the shifted linear gradient velocity model f).

its center). The receiver geometry consists of 35 stations positioned on a regular grid spaced 10 and 20 km approximately along longitude and latitude, respectively (Figure 2). This station geometry results in an uneven distribution of source-receiver distances (i.e., increasing number of receivers with distance; Figure 3).

[15] The travel times are calculated in a 1D model with a linear gradient of 0.1 km/s/km in depth. The travel times for the forward problem have been determined using the ray tracer of *Um and Thurber* [1987]. No noise has been added to the synthetic travel times because we have chosen to appraise the technique when only the velocity structure is varied.

[16] The DD inversion has been repeated with two velocity models differing from the "true" model used to generate the synthetic data-a layered structure and a "shifted" velocity gradient similar to the "true" one but with velocity values 0.3 km/s smaller (Figure 2).

[17] In the following discussion we will address the effect of the velocity structure alone using the complete data set, and using various depleted station geometries. Unless specified, the initial position of the foci has been set to their true position or to the position obtained from the standard, single-event location. For each inversion, we determine the data misfit (as standard deviation) from the re-determined double-difference after the perturbations to the hypocenters have been applied and the travel times recalculated.

[18] The layered velocity model adopted in the application of the DD technique consists of three layers with velocities of 4.4 km/s from the surface to 9 km depth, 5.6 km/s from 9 to 19 km and 6.5 km/s as basement velocity (Figure 2).

[19] In the upper row of Figure 3 we show the z partial derivatives (vertical slowness of the rays at the hypocenters) plotted versus distance for the "true" gradient model used in the generation of the synthetic arrival times (filled light gray circles) and for the layered model used for the DD inversion (black solid circles). On the left panel of Figure 3 we emphasize the differences in value of the vertical partial derivatives by representing the distribution as a histogram (the gray outline refers to the "true" vertical partial derivatives). Overall, we find a remarkable difference in vertical slowness values which enter into the elements of the matrix of equation (4).

[20] The DD inversion has been repeated using the entire data set (i.e., 7350 double-difference equations) and with several subsets (i.e., stations beyond 10, 20, 30 and 40 km, within 30 km, and in the distance ranges 20–35, 30–45 km). Each inversion seeks the solution for a total of 84 (4×21) unknown adjustments (the center earthquake location adjustments are also sought but its position is heavily conditioned).

[21] Because the cluster is relatively small, we have allowed for all the possible links between the foci, that is, each hypocenter is linked to all the other 20. A total of five iterations were performed, although the adjustments tended to zero already by the second iteration. The condition number of the entire mapping matrix A equals 373, which insures a reasonable degree of independence among the rows of the matrix and, thus, the system of equations determinacy. This insures that, regardless the foci initial position, the same final relative location is obtained.

[22] The results of the inversion with the complete data set and constraining the correct position of the center event are shown as solid circles in the top panel of Figure 3. We see that the foci move from their initial positions; the circular distribution of foci is flattened significantly in depth. The root mean square (RMS) of the (DD) residual times decreases from 0.027 to 0.016 s for a decrease of nearly 41%.

[23] The inversion with the depleted data sets shows that by removing progressively the closest stations, the DD

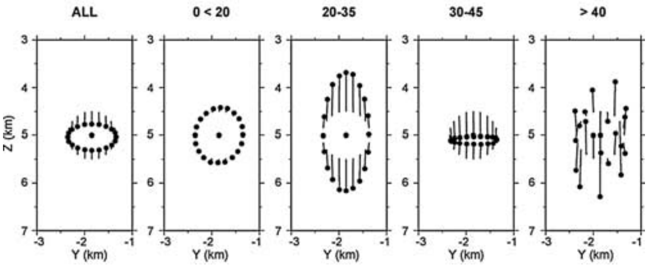

Figure 4. Results obtained with the layered velocity model and receivers from different distance ranges. The thin lines connecting the relocated foci help to visualize the DD-determined adjustments.

inversion results exhibit an increase in the degree of flattening of the original circular cluster. The solution obtained including only the stations beyond 30 km results in a shortening to 30% of the original radius vertically. The RMS of the residual times decreases by about 63% after application of DD in all three cases.

[24] In contrast, the DD relocations obtained using only the closer stations (i.e., 0–30 km) result in an elliptically-shaped cluster of foci with major-axis aligned along the vertical direction. The RMS residual misfit decreases here by nearly 80%.

[25] Similar results, in terms of relative foci locations within the cluster, were obtained when the initial position of the events was set to that obtained from standard location using the velocity model adopted by DD. However, we have found that, in terms of per-cent of RMS residual misfit, all these inversions show reductions somewhat smaller than those in which the reference event is fixed to its true position. This is due primarily to the lower (about 30%) initial data misfit of the standard, single-event locations used for the starting locations in DD.

[26] In the bottom panel of Figure 3, we present the results obtained using the "shifted" gradient model. In this case, because the gradient is similar to the "true" model, even though the velocities are smaller, the cluster of earthquakes preserves its circular shape but it decreases in size. This is a result of the smaller velocities or, equivalently, the larger vertical slownesses shown in the center panel of the lower-row of Figure 3. Remarkably, we find that the same relative location results are obtained regardless of the data subset that is used. This is due to the overall similarity in vertical slownesses between the true and the "shifted" gradient model for rays at all distances for the source-receiver geometry. In addition, this test shows that, in order to asses the robustness of the relative location results obtained with DD, it may suffice to repeat the inversion, adopting different source-receiver geometries for the same set of earthquakes, and check the stability of the results.

[27] Finally, in Figure 4 we further investigate the dependence of the DD relocations on the data subset. We show that the pattern of relative event relocations for the layered model case varies significantly when stations within different epicentral distance ranges are used. The relocated foci align horizontally with the distance range 30–45 km. The condition number for this range of distances is 470 which indicates a good solution; this demonstrates that horizontal streaks of foci can result solely by inappropriate choices of the velocity model combined with particular source-receiver geometries. Conversely, for epicentral distances larger than 40 km the condition number increases to very large values and the solution is inherently unstable, as is shown by the irregular distribution of the relocated foci.

4. Conclusions

[28] Our simple but realistic tests made with synthetic travel times demonstrate that application of a relative location technique such as DD with an incorrect velocity model will give erroneous values of partial derivatives in the mapping matrix. This will, in most cases, lead to bias and error in the relative locations of closely spaced events. In our test, this effect is well shown by the ellipsoidal shapes of the true circular pattern of foci after application of DD. Thus the use of a double-difference technique for high resolution studies of closely spaced events does not preclude having a good velocity model for the whole study region.

[29] This study indicates also that the robustness of a double-difference inversion should be appraised using different data subsets and different velocity models for the same set of events.

[30] **Acknowledgments.** This research has been partially funded through GNV and GNDT contracts. The GMT graphics software *Wessel and Smith* [1991] has been used for some of the figures.

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A. Lomax, Anthony Lomax Scientific Software, Mouans-Sartoux, France. (anthony@alomax.net)

A. Michelini, Istituto Nazionale di Geofisica e Vulcanologia (INGV), Roma, Italy. (michelini@ingv.it)